Discovering mathematical phenomena in primary school mathematics lessons

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Abstract

This study examines the discovery work of a pair of primary school students occupied with a mathematical assignment. It describes how third-grade students make their discoveries about "arithmetic triangles," a task format that is designed for the educational purpose of enabling the recognition of arithmetic patterns and structures (e.g., number relationships). By focusing on how students' work is interactively accomplished, I show what is constitutive of this work: its *material embeddedness*, manifested in a coordinated interplay of domain-specific epistemic practices (the practices of noticing, referring, and connecting) and the visual features of the arithmetic triangles as specific objects of school mathematics, and its *situatedness* in the institutional and social setting of the classroom.

INTRODUCTION

Qualitative social research has so far mainly been concerned with "discovering" in science. There is a long tradition of laboratory, science, and science and technology studies that focus on how scientific work is conducted within socially situated laboratory and field practices. In particular, social researchers have investigated how scientific facts are socially constructed in chemistry and biochemistry (Knorr-Cetina 1981, 1999; Latour and Woolgar 1979; Law and Williams 1982; Mulkay and Gilbert 1982), what routine lab practices in the neurosciences and molecular biology consist of (Lynch 1985a, 1985b; Lynch and Jordan 1995), and how a scientific discovery is locally produced in astronomy (Garfinkel, Lynch, and Livingston 1981), physics (Sormani 2014), archaeology (Goodwin 1994, 2000), and oceanography (Goodwin 1995). While social researchers have been preoccupied with highlighting the mundane methods of producing scientific discovery, more "mundane" discoveries, such as those in educational settings, have been overlooked. This study examines the discovery work of school students engaged in a mathematical task. It describes how third-grade students make their discoveries about "arithmetic triangles," a task format that is expected to enable the recognition of arithmetic patterns and structures (e.g., number relationships). This discovery work differs in several aspects from what can be considered a "scientific discovery."

In fact, what students are supposed to do is neither to invent something new nor to reproduce something that others have discovered. The students' discovery work is set up as a school task designed to learn mathematical phenomena, not to deal with mathematics as a science. Furthermore, it is designed as a task format that deals with specific objects from school mathematics: arithmetic triangles, with which the students in the present study are concerned, do not represent any mathematical objects that professional mathematicians deal with. They are didactically invented school objects specifically designed for educational purposes. In this respect, the aim of the students' discoveries is also not to re-discover something that others, such as professional mathematicians, already know as representing scientific facts. On the other hand, students' engagement with these invented objects also makes it possible to learn some phenomena of mathematics as a science, such as number relations and ways of thinking and speaking mathematically. In this sense, the students also *do* discover something new—something new to themselves.

The aim of the present paper is to describe what the practical and interactional work of students consists of when they make their discoveries about arithmetic triangles as specific objects of school mathematics. The analysis is organised around a detailed case of a pair of students working on an assignment on arithmetic triangles. Adopting the perspective of ethnomethodology and conversation analysis, I analyse, on the one hand, the ways in which the students arrive at their "findings," and, on the other hand, the specifics of the interactive organisation of students' discovery work. Concerning the first aspect, I show that what school mathematics objects allow one to discover is neither a result of one's specific cognitive abilities nor just a feature of these objects themselves. Although arithmetic triangles can be considered, to borrow a formulation of Sharrock and Anderson (2011, 47), objects "with an inbuilt, discoverable organisation," their discoverability is an *interactional phenomenon* grounded in a coordinated *relation* of domain-specific epistemic practices (the practices of noticing, referring, and connecting) and the visual features of these objects. The second argument concerns the social and institutional setting in which the students' discovery work is situated. The framing of this work as a school assignment has implications for how the discovery is accomplished and what the participants understand as discovery. As I will show, students' discovery work is constituted as an activity structured by its orientation towards the *completion of a task*; and, as a task carried out with and among others in the classroom, it is characterised by a specific *sociality*.

PREVIOUS WORK ON DISCOVERY PRACTICES IN SCHOOL SETTINGS

There is a large body of research addressing issues of mathematics and science education (e.g., Gilbert 2005; Greeno and Goldman 1998; Marckwordt et al. 2022; Mercer et al. 2004; Sadler, Romine, and Topçu 2016; Simon et al. 2006; Watkins and Manz 2022) and inquiry-based teaching and learning in particular (e.g., Aulls and Shore 2008; Forbes 2011; Ford 2008; Hammer 1997; Jaber and Hammer 2016; Miller et al. 2018; Roth and Bowen 1994). These studies highlight different aspects of what can be considered important for science learning at school, how students can develop discipline-specific explanations and build scientific knowledge, and how teachers can effectively support them in doing so. Most of these studies are primarily concerned with the conceptualisation, implementation, and investigation of specific teaching and learning methods and frameworks aimed at facilitating students' engagement in science talk and understanding of scientific concepts. In contrast, my focus is on an ordinary, everyday mathematics lesson in a "traditional" primary school classroom, where neither an inquiry-based teaching model nor specifically designed pedagogical methods have been used to promote mathematics learning. With this analytical focus, I draw primarily on ethnomethodological and conversation analytic studies that investigate how everyday classroom practices are routinely produced, maintained, and make sense to those involved.

Classroom interaction and instructional practices have been analysed intensively from an ethnomethodological and conversation analysis perspective (e.g., Hester and Francis 2000; Jakonen 2020; Macbeth 1990, 1992, 2004, 2010; Margutti and Drew 2014; McHoul 1978; Mehan 1979; Moutinho and Carlin 2021; Niemi and Katila 2022), showing how classroom activities are interactively organised and how "instruction-in-interaction" (Lindwall and Ekström 2012) is collaboratively produced using a variety of interactional resources. In particular, recent studies have demonstrated that not only talk but also embodied conduct—such as gestures and touch—and material artefacts, alongside spatial and temporal arrangements, are central to instructional practices in the classroom (e.g., Bergnehr and Cekaite 2018; Cekaite and Dings 2023; Jakonen 2020; Kääntä and Piirainen-Marsh 2013; Lindwall and Ekström 2012; Majlesi 2018; Tanner and Sahlström 2018; Tyagunova and Breidenstein 2023). In recent years, several ethnomethodological and conversation analytic studies have also focused on how not only teachers' instructional activities but also students' learning practices are interactively accomplished in both whole class discussion and student group work phases (e. g., Kämäräinen et al. 2021; Koschmann 2013; Lindwall and Lymer 2011; Melander 2012; Sahlström 2011; Sherman and Tůma 2023; Wakke and Heller 2022). While these studies have highlighted a variety of aspects integral to teaching and learning activities in the classroom, there are only few that have focused on issues of discovery practices in mathematics and science learning in school settings (Ford 1999; Lindwall and Lymer 2008, 2011; Lynch and Macbeth 1998; McHoul and Watson 1984; Zemel and Koschmann 2013).[1]

Lynch and Macbeth's (1998) study on a science demonstration in two primary schools' physics lessons is of particular interest for the present paper. Their focus is on how classroom science is ordinarily produced—that is, how some elementary physical phenomena and processes are presented, enacted, and made discoverable by the teachers to the audience of students. To characterise these demonstrations, the authors speak of a "science spectacle" and

1 However, there are some ethnomethodological studies that have investigated the "work of doing mathematics" by professional mathematicians (Greiffenhagen 2008, 2014; Greiffenhagen and Sharrock 2011; Livingston 1986), showing the inevitably situated and material nature of mathematical practices. See also Greiffenhagen and Sharrock's (2008) critical discussion of Jean Lave's (1988) work on "everyday" mathematics and its relation to "school" mathematics.

describe the practices they identify as aimed at establishing and maintaining what they call the "disciplined witnessing" of the observed phenomena. With their gestural and discursive instructions, the teachers set up a "phenomenal field" (ibid., 277) of perception through which the students are supposed to witness the presented materials and actions as material representations of certain physical phenomena (in that case, molecules) and thus develop a disciplined way of seeing. Another set of teacher activities described by Lynch and Macbeth is aimed at encouraging students' explanations of what is observed and seen. Lynch and Macbeth show how students are repeatedly prompted to collectively witness the scientific spectacle performed before their eyes and describe what they observe in scientific language. The main argument that the authors make at the end of their analysis is that the common criticism from education researchers of classroom science demonstrations as "mock-ups" able only to "make false provision for the situations of inquiry in which professional scientists work" (ibid., 273) relies on an "extrinsic standard of comparison" (that of professional science) and thus misses the specific "integrity and autonomy" (ibid., 288) of classroom lessons that differ from situations of professional science in their tasks and "various organizational contingencies inhabiting the school setting" (ibid., 273).[2]

I share this perspective on classroom lessons here, and many of the teacher practices described by Lynch and Macbeth can be also found in my data. However, my primary focus is not on the teachers' instructive actions but on the students' discovery activities. What the students are supposed to do in the lesson that my analysis focuses on differs in some respects from what the students do in Lynch and Macbeth's lesson. Their role is not just to witness and articulate what the teacher demonstrates to them but to discover things for themselves with very little guidance, as the teacher consistently holds back her own explanations and corrections of the students' explanations. This tendency, alongside the fact that the students are encouraged to work collaboratively in pairs, places different demands and expectations on them than in situations of collective witnessing and explaining.

The studies of Stevens and Hall (1998) and Lindwall and Lymer (2008) on "disciplined perception" are of further relevance to the present paper. The former use the notion of disciplined perception to emphasise two interrelated aspects: first, that disciplines have specific visual practices, "a set of specific forms of embodied action" related "to the tasks, artifacts, and settings where they are deployed" and, second, that there are "interactional and organizational means through which disciplined perception is learned" (Stevens and Hall 1998, 110). Consequently, the focus of their study is on how participants learn to see discipline-specific phenomena or, as Lindwall and Lymer put it, what "cognitive and practical competencies" are involved in "producing, recognizing, and understanding" these phenomena, as well as what characterises the "interactive work by which these competencies are made into

2 As Lynch and Macbeth note with regard to the lesson they analyse: "The students and teachers made use of and concertedly developed ways of describing the equipment and materials, what they were doing with them, and how they were related or unrelated to science. Aside from how well or badly such vernacular descriptions exemplify professional science, they were comprehensible and comprehended well enough to achieve them by the performers and witnesses of the demonstrations" (1998, 289).

objects of learning and instruction" (2008, 190). The insights from these studies are illuminating for my analysis in that they show the importance of *visual orientation* and *coordination* for exploring mathematical and scientific phenomena. In particular, Stevens and Hall, who analyse the interaction between an eighth-grade student and his adult tutor during six tutoring sessions on mathematics learning, demonstrate how the orientation towards visual aspects of the linear functions on the Cartesian plane and their coordination with quantities are "the primary means through which the student "figures out" the task" (1998, 113). As Stevens and Hall note (ibid., 143), this approach distinguishes the student's way of using Cartesian space from that used in the discipline of mathematics, where calculations from equations are usually the primary resource on which professional mathematicians draw.

Finally, in their analysis of how two groups of middle school students solve mathematical problems in a virtual environment, Zemel and Koschmann illustrate the constitutive role of "referential practices," or "the ways that actors refer to and represent problems and solutions" (2013, 66). The students' "discovering work" on the problems they faced involved, as the authors put it, "recalibrating referential practices" that consisted of "the pursuit and production of greater referential specificity or granularity" (ibid., 81), and it is this "work of specifying the indexical properties of unknown things that allows what was previously unknown to become known" (ibid., 83). There are certain similarities between the students' discovery work in my study and that in Zemel and Koschmann's article, in that in both cases practices of pointing and referring to constituent elements and the features of the objects the students are working with play an important role. However, there are also some significant differences, as the interactional contexts in which the students work are essentially different. First, in contrast to the students in the present study, whose work is organised in pairs in a traditional classroom, the students in Zemel and Koschmann's study work in an online environment. This situation imposes specific constraints on communication and achievement of a shared understanding, as the geographically dispersed students are offered only two interaction spaces, a chat and a virtual whiteboard. Since chat postings and whiteboard drawings cannot be accomplished simultaneously, the students' interaction encounters the specific problem of relating these different representations to each other. Second, in contrast to the present study, Zemel and Koschmann's students' work on solving mathematical problems was not a part of the regular school curriculum. It was framed in terms not of learning but of displaying the best collaboration strategies in approaching a mathematical problem. Considering that the local circumstances in which an activity is situated are relevant features towards which participants are oriented, the question I am interested in here is: What characterises the students' discovery work when framed as a *learning* task and done with and among others in the classroom during an ordinary primary school mathematics lesson?

DATA AND ANALYTICAL APPROACH

The empirical material presented in this article originates from a larger research project that investigates the relationship between the interaction order of classroom lessons and their orientation towards enabling subject-specific learning. Considering that classroom interaction establishes its own order—"it has a life on its own and makes demands on its own behalf" (Vanderstraeten 2001, 273)—and that the emphasis on the progression of interaction ("keeping the interaction going"—Koole 2012, 1913) can take priority over the subject-specific problems of learning, the research question that motivates the project is twofold. The analytical attention is directed, first, to the description of the interactive conditions under which school learning is situated and, second, to the analysis of what constitutes the specificity of the subject-related dimension of classroom interaction. As part of the project, a large standardised video study was conducted in which mathematics and first language lessons (one hour each) in 20 German primary school classes in grade three were videotaped.[3] In both lessons, in addition to periods of whole class discussion, there were longer phases of collaborative work in which students worked with a partner on their materials (mostly paper worksheets). In terms of the content to be taught, the mathematics lessons the present paper focusses on were structured around ways of solving and discovering arithmetic patterns and structures. For this purpose, the teachers used two established task formats: number walls and arithmetic triangles.

The video data were collected using a multi-camera strategy: in addition to a teacher camera placed at the back of the classroom and a student camera positioned in a front corner, up to 13 action (GoPro) cameras were installed. These cameras were mounted in the middle of the students' desks to capture the students' interactions with each other and with the materials on their desks. The entire corpus of video recordings consists of almost 370 hours of videotaped interactions. For the purposes of this article, a mathematics lesson was selected for detailed analysis, in which eight groups of two students each were working on an assignment on arithmetic triangles. From this lesson, the video recording of the work of one student group was chosen to examine more closely how the students accomplish their discovery of arithmetic triangles. The episodes that I present and analyse below document the phase of the lesson in which the two students, working as a pair, deal with the first task of the assignment given to the class by the teacher.

The selection of these episodes was motivated by the specific, self-explicating character of the students' discovery work in the videotaped scene. Although, as ethnomethodological studies have shown, self-explication is a feature of many social practices in the sense that they make themselves recognisable or "accountable," as Harold Garfinkel (1967, 37) once postulated, some can be considered to have a specific explicitness that goes beyond this. In this context, Melvin Pollner speaks of "explicative transactions" and uses the example of municipal traffic courts to demonstrate how participation in the proceedings conveys the sense and meaning of necessary actions to defendants arriving in the setting without a prior understanding of how to behave: they are "explicated in the course of their being accomplished

3 The data were collected in the period 2022–2024 by the research training group Subject Specific Learning and Interaction in Primary School (INTERFACH) of Martin Luther University Halle-Wittenberg and the University of Kassel.

and witnessed" (1979, 227).[4] I use the notion of "self-explication" not to refer to specific "self-explicating settings" as Pollner does but to highlight the particularly pronounced self-explicating character of the actions of one group of students in the lesson under study. In the videotaped scene, it is the self-explicitness of the ways of saying and doing—the ongoing production of explicit formulations and pointing by one of the students—that makes this scene especially favourable for this kind of analysis. Compared to the other student desks videotaped, the selected scene shows in a particularly explicit way what most of the other students in the selected lesson do when they discover arithmetic triangles.

The analysis is grounded in ethnomethodological "studies of work" (Garfinkel 1986) and the micro-ethnographic and conversation analytic approach of embodied interaction (Goodwin 2000, 2018; Streeck 2009, 2010). When analysing the video sequences, I focus on how students orient themselves to the assignment and to each other's actions as they become involved in discovering. The use of the video-based data is particularly revealing of not only what participants, in the unfolding interaction, are actually doing but also of how they understand their discovery work. It also makes it possible to describe in detail the embodied aspects of students' discovery work, as this work, like many other social practices that we find in everyday and professional settings, is characterised by thoroughgoing materiality and multimodality (cf., e.g., Cekaite and Mondada 2020; Haddington, Mondada, and Nevile 2013; Heath and Luff 1992; Nevile at al. 2014; Suchman 1987). The episodes presented have been transcribed according to the conventions of multimodal conversation analysis (cf. Mondada 2018).

ANALYSIS

The Lesson and the Task

Before proceeding with the analysis, I provide a brief overview of the lesson and assignment. As previously described, the students were working on a mathematical assignment about arithmetic triangles. When introducing the lesson topic and the assignment, the teacher used the terms "maths research" and "solving a puzzle" to describe what the students were supposed to do in the lesson.

From a mathematics education point of view, arithmetic triangles represent a task format that enables, on the one hand, the practice of such basic arithmetic skills as addition and subtraction and, on the other, the recognition of arithmetic patterns and structures, such as number relationships and interrelationships between numbers (Krauthausen and Scherer 2022, 140). Arithmetic triangles are treated as a form of a "substantial learning environment" (Wittmann 2001, 2)—that is, as a teaching unit that allows numerous tasks with different content- and process-related goals (calculating by addition and subtraction, exploring, reasoning, arguing) at different difficulty levels (Krauthausen and Scherer 2022, 140f.; Wittmann 1995, 365–366). Arithmetic triangles, among other substantial task formats, are

4 See also Livingston (2006) on the self-explicating and self-elaborating character of the practices of reading.

characterised by the fact that they always have the same basic form and always contain a constant rule, introduced in the lesson, that can be used to formulate different problems to be solved.

The basic form of an arithmetic triangle is an equilateral triangle with three inner and three outer fields in which numerical values are entered. The simple rule is as follows: the number of the respective outer field (outer number) corresponds to the sum of the numbers of the adjacent inner fields (inner numbers). For example, the sum of the inner numbers 7 and 2 results in the outer number 9 (fig. 1).

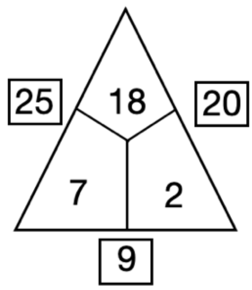

Figure 1. Arithmetic triangle

The studied lesson consisted of several phases in which the students worked through a sequence of tasks issued by the teacher. The first phase was designed primarily to be a review: the students, together with the teacher, reviewed the terms "inner numbers" (German: *Innenzahlen*) and "outer numbers" (German: *Außenzahlen*) and practised solving an arithmetic triangle where all the inner numbers were given by calculating the outer numbers for such a triangle. This phase resulted in the following picture on the blackboard (fig. 2):

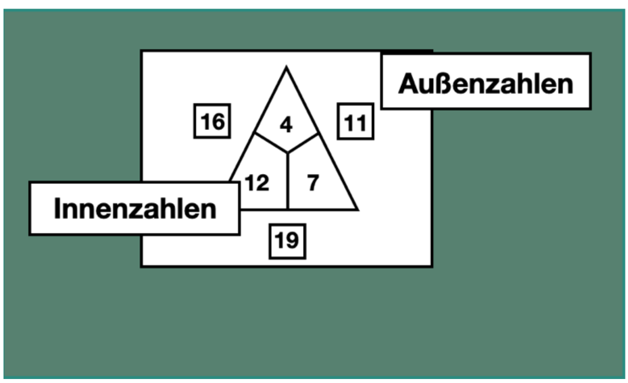

Figure 2. Arithmetic triangle on the blackboard to practise

The task for the following phase, which was designed as partner work and is the focus of the current analysis, was formulated as follows: solve the arithmetic triangles, find out what the next two triangles must look like, and write down what you notice. While formulating this task, the teacher showed a worksheet and pointed to the blackboard where the sequence of task steps for the lesson was illustrated (fig. 3). There were five arithmetic triangles to solve on the worksheets that the students received for this first task: three in which all the inner numbers were given and two in which no numbers were given (fig. 4). The students were thus required to find missing numbers in the two "empty" arithmetic triangles on their worksheets by solving and discovering the first three arithmetic triangles with the given inner numbers. During this phase, the teacher was continually moving around the classroom, from one student desk to the next, giving the students procedural advice and encouraging them to describe what they noticed about the arithmetic triangles and, in some cases, to explain to her or to their partner how they had found the missing numbers.

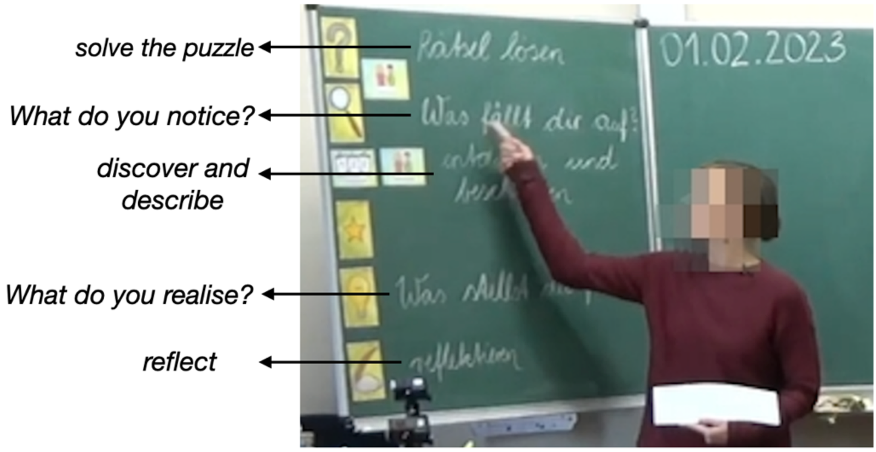

Figure 3. Sequence of task steps for the lesson

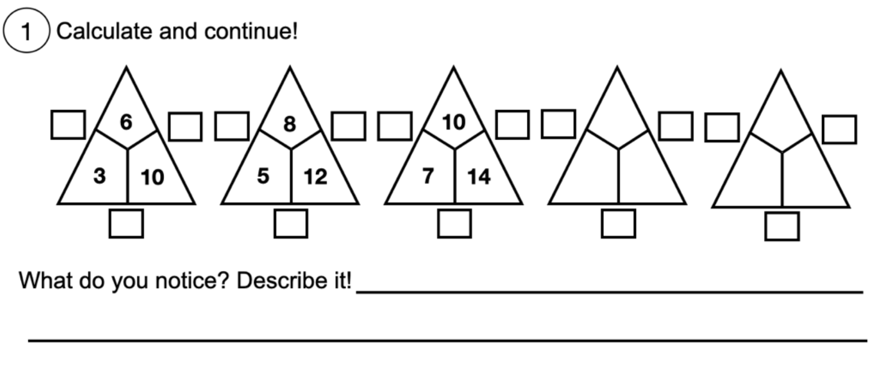

Figure 4. Worksheet for task 1

In the following phase, organised as a whole class discussion, the students and the teacher discussed what the students had "noticed" when "solving the puzzle," and they produced a rule describing the relationship between the inner and outer numbers in these particular arithmetic triangles. This activity was followed by another phase of partner work in which the students were asked to solve further arithmetic triangles with the goal of discovering, describing, and realising a pattern, namely that the increment of change in the outer numbers is always double said increment of the inner numbers. The results of this phase were also the subject of the subsequent whole class discussion.

In the following section, I consider in detail how the discovery work of the first task (cf. fig. 4) was enacted by a pair of students and how they arrived at their findings.

"Reading from" the Structure: A Number Sequence

The excerpts below are organised around a series of steps that can be observed in the students' work on the task in the analysed scene. The first excerpt is particularly revealing in terms of the relationship between the structure and the visual aspects of the material on the one hand and the verbal and embodied practices of explaining what is seen on the other.

EXCERPT 1

01 Sm02: +Hey, was kommt# denn jetzt hier?
Hey, what's coming here now?
sm02 +points to empty Δ-field, looks at Sm18, at his ws
fig #Fig. 5
02 (.)

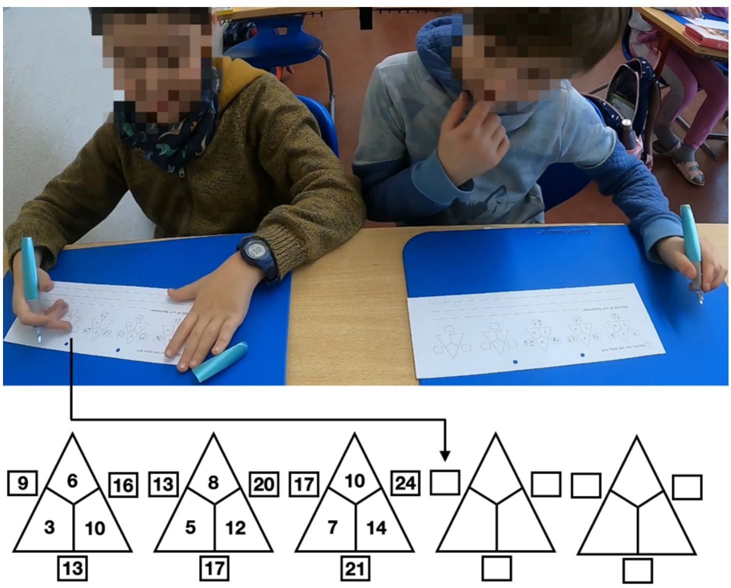

Figure 5

03 Sm02: +[Oh!
Oh!
sm02 +looks at his ws

04 Sm18: •[Ausdenken!
Think it up!
sm18 •looks at Sm02
05 Sm02: +Nein. Etwas herausfinden.
No. Find something out.
sm02 +shakes his head, looks at Sm18
06 +(3.0)
sm02 +looks at his ws
07 Sm02: Ach so:! +Sm18• guck mal
I see! Sm18 look
sm02 +looks back at Tf, at Sm18, at his ws
sm18 •looks at Sm02, at Sm02's ws
08 +#hier, plus zwei ist gleich zwölf.
here, plus two equals twelve.
sm02 +points to "10" in first Δ, to "12" in second Δ
fig #Fig. 6
09 +#Plus zwei ist gleich vierzehn. (1.5)
Plus two equals fourteen.
sm02 +moves the pen from "12" in second Δ to "14" in third Δ, looks at Sm18
fig #Fig. 6

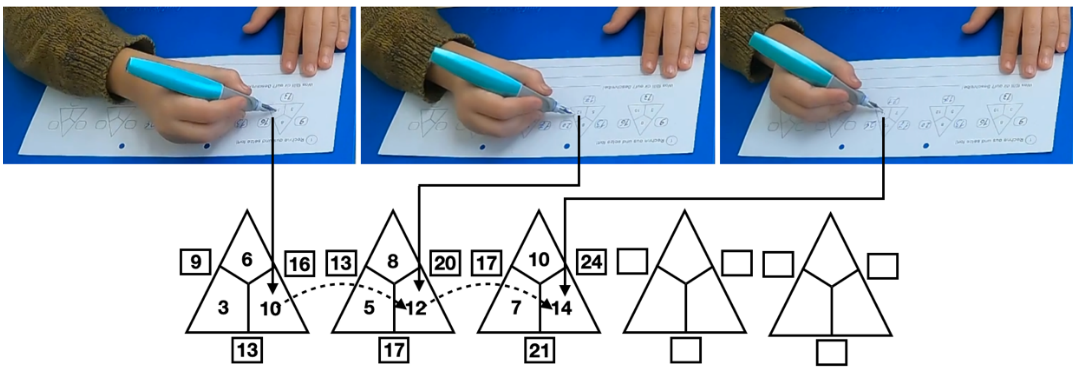

Figure 6

10 Sm02: +Also kommt hier +#sechzehn.
So here comes sixteen.
sm02 +points to the empty bottom right field in fourth Δ
sm02 +writes "16"
fig #Fig. 7

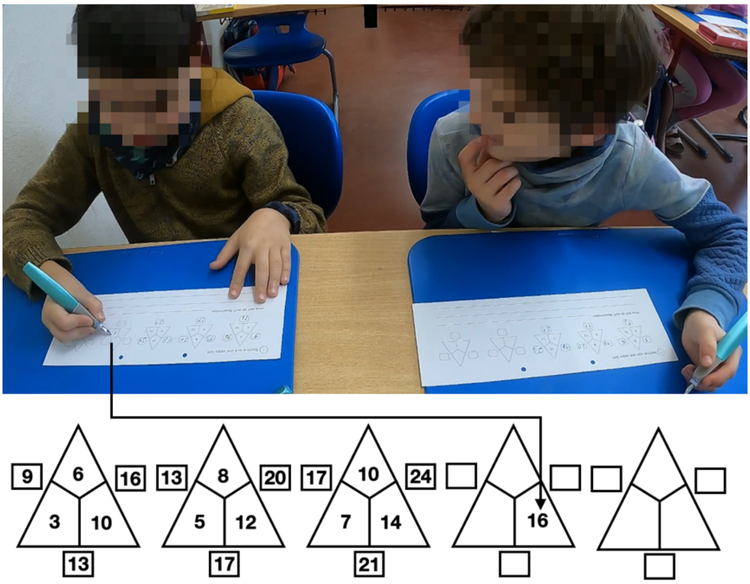

Figure 7

11 Sm18: •Ja::.
Yes.
sm18 •looks at his ws, writes “16”
12 (.)
13 Sm02: Ach so::! Guck mal. Sm18. Hier
I see! Look. Sm18. Here.
14 +#Plus zwei, plus zwei, plus zwei.
Plus two, plus two, plus two.
sm02 +moves his finger from one number to another in upper Δ-fields
fig #Fig. 8

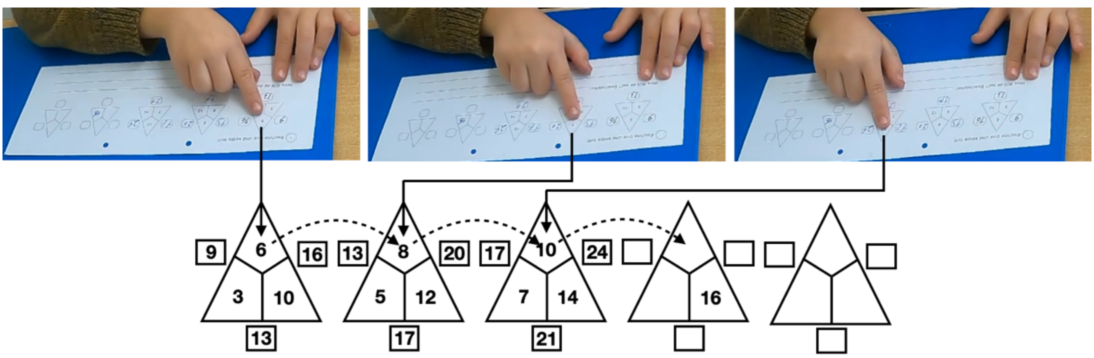

Figure 8

15 +(2.0)
sm02 +writes “12” in upper empty field in fourth Δ

```
16          (2.0)
17          +Plus zwei, plus zwei, plus zwei.
             Plus two, plus two, plus two.
   sm02     +points to numbers in bottom left Δ-fields
18          +#(.)
   sm02     +writes “9” in bottom left field in fourth Δ
   fig       #Fig. 9
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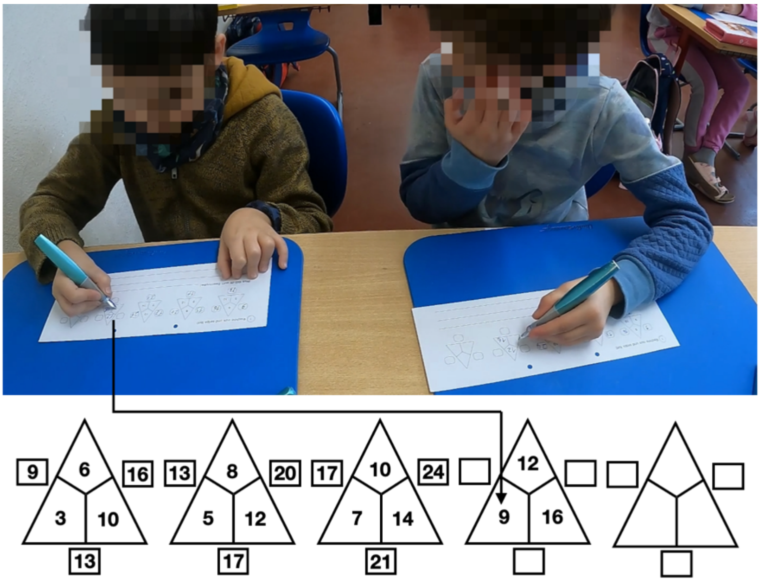

Figure 9

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19 Sm18:    •Neun.
             Nine.
   sm18     •looks briefly at Sm02, writes “9”
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Although “Oh!” (line 3) is somewhat more ambiguous than “I see!” (line 7), both utterances can be understood here as expressions of discovery. The fact that “Oh!” and “I see!” are heard in this way is due to the sequential environment in which they appear—here, in light of Sm02’s previous question (“Hey, what’s coming here now?,” line 1) and his looking at his worksheet (lines 3, 6), as well as in the context of the task of finding out what the next two “empty” arithmetic triangles must look like, which the student explicitly re-establishes in response to his partner’s “wrong” suggestion (line 5). By saying “I see!,” Sm02 claims that he has figured something out and invites his partner to witness this as he continues: “Sm18 look” (line 7). The interactional work that “I see!” does here therefore consists of Sm02 trying to catch his partner’s attention and, at the same time, claiming that he now knows what he did not know before, namely how to find the “missing what” of the last two arithmetic triangles. The question I focus on in the following discussion is how this “missing what”

(Lynch and Macbeth 1998, 281) comes to be discovered. In terms of the task's goal, finding the missing numbers in the two "empty" arithmetic triangles—or rather finding the rule for filling in the missing numbers—is crucial for discovering an arithmetic pattern that describes the number relationships within the given set of arithmetic triangles. The first transcript sequences are thus about how the students come to find the rule.

The claim of discovery is followed by an explanation of what has been found, and, as we will see, the way in which the explanation is accomplished appears to be constitutive of the discovery, for which the structure of the arithmetic triangles provides an important visual resource. The explanation articulates the arithmetic operation (addition) by which certain numbers are related to each other ("…plus two equals…," lines 8–9), but its articulation is materially and visually tied to the visual properties of the arithmetic triangles on the worksheet. While articulating his explanation, the student relates certain numbers in certain positions to each other, which he simultaneously looks at and points to. Pointing gestures and deictic terms like "here" (lines 8, 10) link the visual properties of the arithmetic triangles with the verbally formulated arithmetic operation—or, rather, they materialise it. One can thus note that "here plus two equals twelve" and "plus two equals fourteen" are anchored to particular inner numbers in the same position in different arithmetic triangles. Moreover, the sentence "So here comes sixteen" (line 10) might be heard as an implication of what previous turns are saying and doing. The student thus articulates a number sequence that is, to borrow a formulation of Stevens and Hall (1998, 142), "read from" the structure of the arithmetic triangles, and what this sequence consists of is to be found within his unfolding saying-looking-and-pointing. The intelligibility of the explanation builds on the coordinated relation between the visual aspects of the arithmetic triangles, the sequential order of pointing gestures, and the trajectory of the finger movements, which together clarify the meaning of the verbal formulations by connecting the number positions ("here") to the number difference ("two").

That the explanation would make little sense without pointing references becomes evident as the discovery progresses. In line 13, we find another claim of discovery by Sm02, which is formulated in almost the same way as the previous one. Again, it is followed by an explanation that indicates, verbally and through pointing, a difference between certain numbers increasing by two ("plus two, plus two, plus two," lines 14 and 17). This time, however, the verbal formulation becomes more elliptical. Here, while the verbal formulation does not explicitly state which numbers are to be related, the sequence of pointing gestures highlights the relevant numbers in the arithmetic triangles as critical for making sense of the explanation. It is only through the student's pointing gestures that it becomes clear that what has been found this time relates to the number difference in the two other number sequences. That is, whereas previously the difference was anchored to the bottom right inner numbers (line 9, fig. 6), this time the difference is related to the upper inner numbers and the bottom left inner numbers (line 14, fig. 8 and line 17, fig. 9). The pointing gestures thus appear to be constitutive of the explanation, as they visualise the sequences of numbers by marking the respective positions of the numbers increasing by two.

Describing the Number Sequence

Before the next excerpt, a series of turns has been omitted, in which the two students find and fill in all the missing inner numbers in the two "empty" arithmetic triangles and Sm02 communicates with his classmates from two other tables, saying that he has already figured out the puzzle and that it is easy. The two students are now in the process of filling in the last missing outer numbers by calculating the corresponding inner numbers and describing what they have noticed during their search: they are about to fulfil the second task requirement on their worksheets.

EXCERPT 2

```
01 Sm02:  +Ich hab das Rätsel schon herausgefunden.
           I've already figured out the puzzle.
   sm02   +looks at Tf at the next table
02        (4.0)
03 Sm18:  •(Ich schreibe) immer zwei dazu.
           (I write) always add two to that.
   sm18   •looks at Sm02
04 Sm02:  Ja.
          Yes.
05        +#(5.0)
   sm02   +writes "Always +2"
   fig     #Fig. 10
```

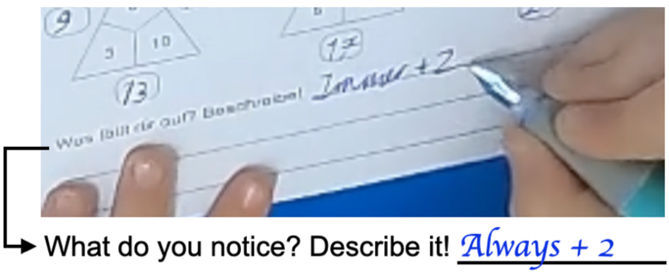

Figure 10

```
06        *(7.0)
   tf     *comes to the table, looks, walks away
07        +•(10.0)
   sm02   +writes
   sm18    •writes
08 Sm18:  Das=äh: (2.0) •vierunddreißig.
          This uh         thirty-four.
   sm18                 •looks at Sm02's ws, at Sm02
```

```
09        (.)
10 Sm18:  [(Ja.)]
           (Yes.)
11 Sm02:  [Nee. ] Zweiunddreißig.•
           Nope.  Thirty-two.
   sm18                          •looks at his ws, writes "32"
12        (3.0)
13 Sm02:  +Hä, das ist so einfach. (2.0) Immer plus zwei.
           Huh, that's so easy.          Always plus two.
   sm02   +looks at the next table
14 Sf19:   Plus fünf- +plus vier.
           Plus five-  plus four.
   sm02               +dismayed expression
15       (.)
16 Sm02:  Nein. +Hier. Guck.
          No.    Here. Look.
   sm02         +takes his ws, looks at Sf19
17       +Plus zwei ist (gleich fünf), plus zwei ist gleich sieben.
          Plus two (equals five), plus two equals seven.
   sm02  +points to his ws
18       (3.0)
19 Sm02:  Fertig. +Sm18, bist du fertig?
          Done.    Sm18, are you done?
   sm02           +looks at Sm18
20 Sm18:  Noch nicht.
          Not yet.
```

The students' description of what they have noticed is interesting in several ways. It makes evident the *collaborative* nature of the work, as it is now Sm18 who, obviously building on Sm02's previous explanations, suggests the description "(I write) always add two to that" (line 3), which Sm02 immediately confirms and starts to write down on his worksheet (lines 4–5). It entails the use of "always" which indicates that there is a *rule* describing the difference, formulated as "add two" (line 3) or "plus two" (line 13), between certain numbers in the arithmetic triangles. Furthermore, the students seem to consider their description sufficient to say what they have noticed and consider their discovery *complete*: first, Sm02 says that he has "already figured out the puzzle" (line 1) and, second, after a short exchange with his classmate from the other table (lines 13–17), announces that he is "done" and asks Sm18 if he is "done" too (line 19). Finally, what has been discovered, noticed, and formulated is commented on by Sm02 as "so simple" (line 13), a recognisably *socially* oriented act addressed to the others in the class, as is Sm02's announcement "I have already figured out the puzzle" in line 1, which, as documented by his gaze direction, is addressed to the teacher in the first line.

The episode thus reveals two things of particular interest about students' discovery work: first, that it is constituted as an activity oriented towards *completing* the instructional task, and second, that this work, as a task accomplished with and among others in the classroom, is characterised by a specific *sociality*. The following episodes show how the work of the two students develops an interesting dynamic in which their orientation towards exploring the arithmetic triangles is tightly intertwined with their orientations towards getting the task done and, at the same time, doing the task as a team.

"Getting Done": Orientation Towards Completing the Task

Excerpt 3 is particularly informative with respect to how the communication between the two students is structured in relation to their efforts to complete the task. Prior to this excerpt, some sequences have been omitted, in which Sm02 has noticed and pointed out a grammatical mistake on Sm18's worksheet and Sm18 has corrected it. Sm18 is still writing the description "Always +2 to that" on his worksheet when Sm02 raises his arm and announces that he is finished.

EXCERPT 3

```
01        +(.)
   sm02   +↑
02 Sm02:  Ich bin fertig.
          I am done.
03        •+(4.0)
   sm18   •writes "to," looks at Sm02
   sm02    +looks at Sm18's ws, ↓
04 Sm02:  +Ich hab einfach immer plus zwei.
           I just have always plus two.
   sm02   +looks at Sm18
05        •+(8.0)•
   sm18   •writes "that", looks at Sm02
   sm02    +looks at Sm18's ws, quick and slightly nodding head movement,
            writes "to that" in his ws
   sm18          •↑
06 Sm02:  Wir sind fertig.+
          We are done.
   Sm02                 +↑
07        •+(11.0)
   sm18   •looks at Tf at the next table
   sm02    +looks at Tf at the next table
08 Sm18:  •Das war leicht.+
           That was easy.
```

sm18 •looks at Sm02 and smiles
sm02 +nods
09 *(6.0)•+
tf *comes to the table
sm18 •↓
sm02 +↓
10 Tf: So was fällt *•+dir jetzt auf?
So what do you notice now?
tf *points to Sm18's ws
sm18 •looks at his ws
sm02 +looks at Sm18's ws
11 Versuch mal noch ein bisschen +mehr zu beschreiben.
Try to describe a bit more.
sm02 +looks at his ws
12 Jetzt hast du immer gesagt, *worauf bezieht sich das dann?
Now you've said always, what does that refer to then?
tf *points to "+2" on Sm02's ws
13 *#Versuch mal +diese beiden •Wörter zu benutzen.
Try to use those two words.
tf *points with her finger to bb
sm02 +looks at bb
sm18 •looks at bb
fig #Fig. 11

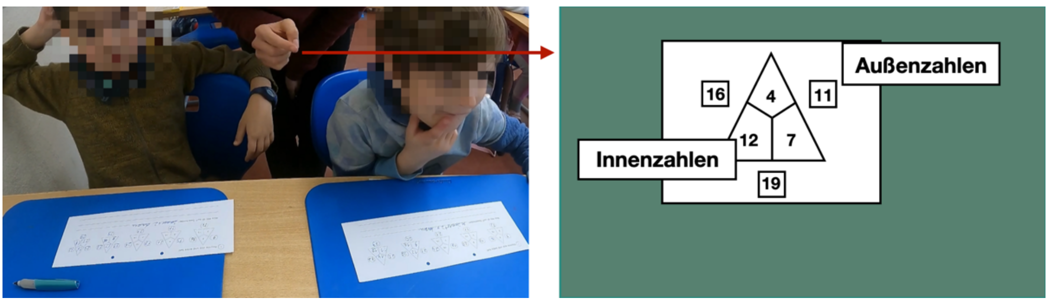

Figure 11

14 (3.0)
15 Sm02: *+Ah::! (Immer plus zwei) dazu. Bei den Innenzahlen.
Ah! (Always plus two) to that. To the inner numbers.
tf *walks away
sm02 +looks at his ws
16 +Be:i de:n Innenzahlen.•
To the inner numbers.
sm02 +looks briefly at Sm18, starts to write

```
    sm18                          •looks at Sm02's ws
17          (.)
18 Sm18:    +•Bei den. Bei (.) den (.) Innenzahlen.#
              To the.  To       the      inner numbers.
    sm02    +writes "to the inner numbers"
    sm18     •writes "to the inner numbers"
    fig                                        #Fig. 12
```

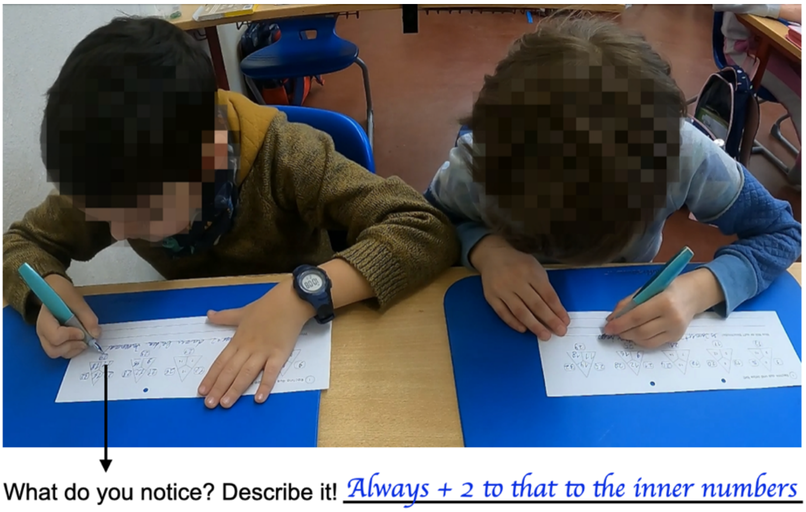

Figure 12

```
19          +•(5.0)
    sm02    +looks at Sm18's ws
    sm18     •still writes "to the inner numbers"
20          •+(2.0)
    sm18    •looks at Sm02's ws
    sm02     +looks at his ws
21 Sm02:    So, +fertig. (.) Punkt.
            So,  done.       Period.
    sm02        +looks at Sm18, knocks on the table, looks at Tf
22          (.)
23 Sm02:    +Jetzt.
             Now.
    sm02    +↑
24 Sm18:    •Jetzt.
             Now.
    sm18    •↑
```

The episode shows once again the collaborative character of the students' work. Although Sm02 has announced that he is finished and has signalled this fact by raising his arm, having observed what Sm18 is writing he puts his arm down and completes his own description ("Always + 2," cf. fig. 10) by adding some of Sm18's wording, in particular "to that" (lines 3–5). This moment highlights an interesting point of Sm02's recognition of Sm18's description when, commenting on the sentence on his own worksheet, he first says "I just have always plus two" (line 4) and then, after observing what Sm18 writes, makes a quick, slightly nodding head movement, as if to signal that Sm18's formulation "Always + 2 to that" is more convincing to him after all. Now, both students display that they are finished by raising their arms and looking towards the teacher, whose attention they are waiting for to show her the completed task (lines 6–8). One may also notice the use of the past tense in Sm18's comment, "That was easy" (line 8), which emphasises once again the students' orientation towards the completion of the task.

However, as the interaction with the teacher in this episode (lines 9–13) and the next (excerpt 4) reveals, the students are still far from having finished the task. Although the teacher seems to consider the students' description to be appropriate ("you've said always," line 12), she points out that it is still not clear what it refers to (line 12) and prompts the students to describe "a bit more" (line 11) using the terms "inner numbers" and "outer numbers —"those two words" (line 13) on the blackboard. The teacher thus indicates that the students' answer to the question "What do you notice?" is not complete because, as becomes especially clear in the subsequent episode (excerpt 4), it is not specific enough in its formulation: it does not bring the rule identified by "always" into a differentiated connection with the inner and outer numbers in the arithmetic triangles.

When the two students follow the teacher's advice and look at the words on the blackboard, the terms "inner numbers" and "outer numbers" (as represented in fig. 11), they can be seen to have turned back to their discovery work. After three seconds of intensive looking at the blackboard, Sm02 says "Ah!" (line 15), preceding his restatement of the description of what the students have noticed, now supplemented by "to the inner numbers" (lines 15–16). Here one can note the similarity between this "Ah!" and the previous "Oh!" and "I see!" (excerpt 1), which seem to have the same function: to claim that something has been found out or realised.

After the students have completed their descriptions by adding "to the inner numbers" and have checked to see if each has finished his description (lines 16–20), they signal once again that they are done with the task (lines 21–24). There is an interesting moment of coordination of actions as a joint activity when Sm02, who finishes his description a little earlier, seems to be waiting for Sm18 by looking at Sm18's worksheet (line 19) and only upon noticing that Sm18 has finished writing too announces, "So, done. Period" (line 21). After that, the two students almost simultaneously raise their arms, saying "Now" (lines 23–24). Thus, although the two students are evidently oriented towards the completion of the task as quickly as possible, they are also trying to do their work as a team.

Pursuing Discovery: Trial and Error Method

The next episode (excerpt 4) takes place immediately after the previous one and shows a further shift in the students' orientation from having the task done to continuing the discovery work. When the teacher comes back to the two students, it turns out that even "now" the task cannot be considered finished.

EXCERPT 4 (CONTINUED FROM EXCERPT 3)

```
25          (22.0) *(2.0)
                   *comes to the table
26 Sm02:    *Jetzt.+
             Now.
   tf       *looks at Sm02's ws
   sm02            +looks at Tf
27 Tf:      Mhm. Jetzt *+denkt mal an die AUSSENzahlen nach.
            Mhm. Now think about the outer numbers.
   tf                  *points with her finger to bb
   sm02                 +looks at bb
28          Was findet ihr •über *die AUSSENzahlen heraus?
            What do you find out about the outer numbers?
   sm18                    •looks at his ws
   tf                            *walks away
29          +(3.0)
   sm02     +looks at his ws
30 Sm02:    Sm18.• (.)+ (.)• (inaudible) minus (.) plus
            Sm18.                        minus     plus
   sm18          •turns to Sm02
   sm02              +knocks briefly on the table
   sm18                   •looks at Sm02's ws
31 Sm02:    minus=plus=minus=plus+
            minus plus minus plus
   sm02                          +looks at Sm18, shows his tongue, laughs
32          +•(5.0)
   sm02     +looks at his ws
   sm18      •looks at his ws
33          +(2.0)
    sm02    +points with his finger at his ws
34 Sm02:    Ach!• (2.0) +#Plus vier (.)
            Ah!          Plus four
   sm18         •looks at Sm02's ws
   sm02                 +moves his finger from "9" to "13"
```

```
   fig                  #Fig. 13
35 Sm02:  +#plus sieben (.)
            plus seven
   sm02   +moves his finger from “13” to “20”
   fig     #Fig. 13
36 Sm02:  +#minus
            minus
   sm02   +points to “17,” looks briefly at Sm18
   fig     #Fig. 13
37        +•(6.0)
   sm02   +moves his finger between the numbers, frowns
   sm18    •looks at Sm02’s ws
```

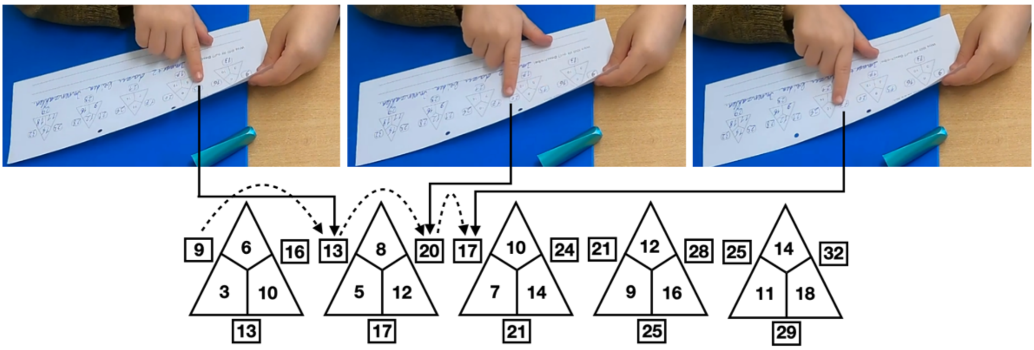

Figure 13

After a short look at Sm02’s worksheet, the teacher produces an acknowledgement token (“Mhm,” line 27), but in the next turn she again draws the students’ attention to the words on the blackboard, saying “Now think about the outer numbers” and pointing to the blackboard. According to the tasks, the students are supposed to write down on their worksheets the descriptions of what they have noticed when they have discovered the arithmetic triangles, and the teacher now makes it clear that this concerns both inner and outer numbers. Since the students have only described the relationship between the inner numbers, and in the light of the teacher’s previous request to concretise the description by using “both” terms on the blackboard (excerpt 3), the students’ work turns out to be incomplete. By asking “What do you find out about the outer numbers?” (line 28), the teacher thus re-establishes the relevance of the further search.

When the students turn back to their worksheets, they seem to have difficulty in identifying what characterises the relationship between the outer numbers. It is again Sm02 who seems to have the first idea when, after looking at his worksheet, he addresses his partner and suggests “minus plus” (line 30). At this point, it is not clear what exactly “minus plus” means. The “minus plus” suggestion seems more like a procedure of trial and error—a strategy that the student tries out first to see if it will work. That he himself is not sure of his idea is displayed both verbally, when Sm02 quickly and somewhat playfully says "minus plus minus plus," and through his laughter and showing his tongue (line 31). After seven seconds of looking at his worksheet (lines 32–33), Sm02 seems to have figured something out, as he says "Ah!," produced as a discovery claim and followed by "plus four" (line 34)—a turn similar to his discovery claim in excerpt 1. However, as he continues, he seems to realise that his idea does not work, because "plus four," "plus seven," "minus," which he marks by simultaneously pointing to the respective numbers, do not describe a number sequence that would show any regularity. This obstacle may be why he does not complete the sentence but simply says "minus" without adding a number (line 36). This failed attempt is followed by another six seconds of Sm02 looking intently at his worksheet, moving his finger between the numbers, and furrowing his brow (line 37).

Seeing the Analogy

After Sm02's first unsuccessful attempt to find out what characterises the relationship between the outer numbers, we see in the next excerpt how he formulates another assumption that turns out to be plausible.

EXCERPT 5 (CONTINUED FROM EXCERPT 4)

38 Sm02: Ah=ja, immer plus vier.
Ah yes, always plus four.
39 +#Plus vier. (.) Plus vier. (.) Plus vier. (.) Plus vier.
Plus four. Plus four. Plus four. Plus four.
sm02 +points to upper left outer numbers
fig #Fig. 14

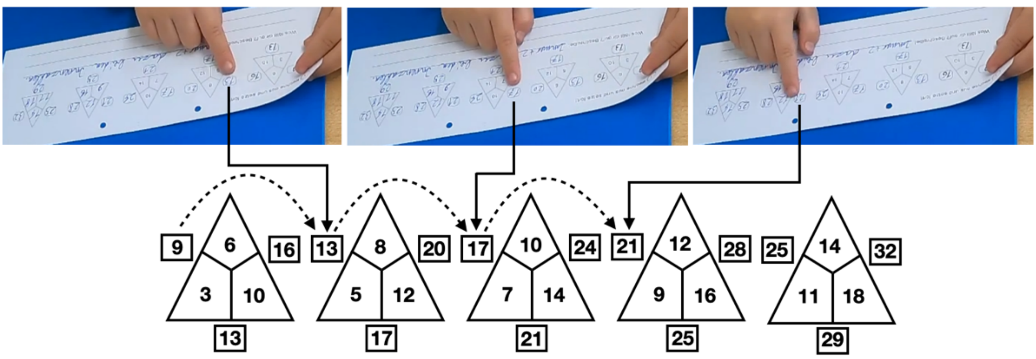

Figure 14

40 Sm18: •Hier ist aber dreizehn.
But here is thirteen.
sm18 •points to "13" in second Δ on Sm02's ws

41 Sm02: Ja, plus vier ist +gleich siebzehn.
Yes, plus four equals seventeen.
sm02 +points to “17” in third Δ
42 +•(3.0)
sm02 +looks at his ws, at Sm18
sm18 •looks at Sm02’s ws
43 Sm02: •Plus vier (.) is +gleich einundzwanzig.
Plus four equals twenty-one.
sm18 •turns to his ws
sm02 +points to “21” in fourth Δ
44 Sm18: Sm02, •hier is sechzehn, aber is doch minus?
Sm02, here is sixteen, but it’s minus, isn’t it?
sm18 •points to “16” in first Δ on his ws
45 (.)
46 Sm02: +Immer plus vier.
Always plus four.
sm02 +looks at Sm18’s ws
47 (.)
48 Sm02: Guck mal rein. Guck mal da.
Look inside. Look there.
49 +Hier sind ja die Zahlen immer dort. Also immer plus vier.
Here the numbers are always there. So always plus four.
sm02 +points to “9” in first Δ and to “13” in second Δ
50 Sm18: Ja das stimmt. Eigentlich stimmt.
Yes, that’s true. Actually true.
51 +•(4.0)
sm02 +starts to write
sm18 •starts to write
52 Sm18: •Aber wir müssen noch die Außenzahlen schreiben.
But we still have to write the outer numbers.
sm18 •looks briefly at Sm02
53 Sm02: Ja, mache ich auch.
Yes, I do that too.
54 Immer plus vier (.) dazu bei den Außenzahlen.
Always plus four to that to the outer numbers.
55 +•(15.0)# (2.0)
sm02 +writes “Always +4 to that to the outer numbers”
sm18 •writes “Always +4”
fig #Fig. 15
56 Sm02: +Fertig. (2.0) Ich bin fertig.
Done. I’m done.
sm02 +↑ and looks at Tf

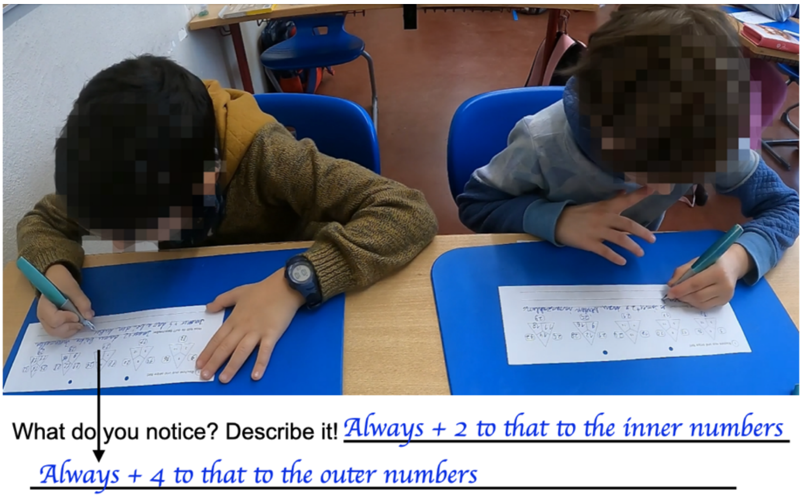

Figure 15

Sm02's utterance in line 38, "Ah yes, always plus four," is similar to his utterance in the previous episode ("Ah! Plus four," excerpt 4, line 34)—it is produced as an announcement of something that has been realised, or, one could also say, as a "'click of comprehension'" (Coulter 1979). However, this time it is heard slightly differently. It is both the similarity to and the difference from the utterance in excerpt 4—the subtle change in the formulation through the adding of "yes" and "always"—that account for this different hearing. By adding "yes" and "always," the student indicates that he was right with his previous suggestion of "plus four" but now he can formulate it as a rule: it is *always* about "plus four." As if to explicate this, he points to the corresponding numbers on his worksheet while simultaneously commenting on his finger movements with "plus four, plus four, plus four" (line 39), an explanation similar to the one he formulated earlier as the rule "always plus two" describing the relationship between the inner numbers (excerpt 1).

This and the following embodied explanations produced by Sm02 in response to Sm18's epistemic confusion (lines 40–50) render clear the problem with Sm02" initial idea in the previous episode (excerpt 4). The rule "always plus four"—like "always plus two" above—does not apply to all the outer numbers but only to those outer numbers, or, in the case of "always plus two," to those inner numbers that have *the same position* in the different arithmetic triangles. This is what Sm02 is trying to make clear to Sm18 when he says "Look inside. Look there. Here the numbers are always there" (lines 48–49), where "there" means "in the same position." This moment is also when Sm02 seems to notice the analogy between the two types of number sequences, as he starts to write down and also explicitly formulates (in response to Sm18's comment, line 52) the rule for the outer numbers through analogy with the rule for the inner numbers. And again, we can see the shift in the students' orientation as, having finished his description, Sm02 hurriedly announces that he is "done" (line 56).

DISCUSSION

The aim of this study was to describe the discovery work of two primary school students occupied with a mathematical assignment during an ordinary mathematics lesson. I have examined the ways in which the students arrive at their findings and explicate what they have discovered about the arithmetic triangles that were the object of their discovery practices. Drawing on the results of my analysis, I will summarise what characterises this discovery work in the investigated lesson and discuss the question of how this work relates to learning.

The Interactive and Practical Work of Students' Discovery

Perhaps the most salient feature of the students' discovery work is its *material* embeddedness. In fact, constitutive of this work is the coordinated interplay of visual, gestural, and verbal practices that relate visual features of the arithmetic triangles to each other and transform them into the "solution of a puzzle." What is considered a solution at this stage of the assignment is finding out the rules that describe the relationships between certain numbers. These rules become searchable and explainable from the material display—the visual organisation of the arithmetic triangles. In a sense, to paraphrase Stevens and Hall (1998), they are *read from* the structure of the arithmetic triangles. However, although finding the rules is, so to speak, built into the structure of the arithmetic triangles, it is an interactional achievement. It is the interactive work of the students through which the rules describing the number relations become visible: their unfolding saying-looking-and-pointing, something that Latour (1986) calls "thinking with eyes and hands." The discovered and explicated number sequences are progressively unfolded as sequences of these sayings and pointings and are based on the selective perception and comparison of the numbers with respect to their position in the arithmetic triangles. The discovered arithmetic rules emerge, to borrow a formulation of Garfinkel, Lynch, and Livingston, as "a locally embedded phenomenon whose 'properties' are come upon in a developing sequence of locally pointed noticings" (1981, 149). There is, in a sense, a didactic moment in the scene, as the students' ongoing verbal and gestural explanations of what they are seeing and doing make visible and disclosable the cognitive and practical competences involved in discovering the arithmetic triangles—the practices of noticing, referring, and connecting.

Although students' discovery work is not "real" scientific discovery, it is to some extent modelled on one. It is prefabricated and, in this sense, a "mock-up" (Atkinson and Delamont 1977), but it still must be carried out as a search (in this case, for pattern-forming rules and regularities), as a combination of observations resulting in finding something "new" for the participants. This work, in the way it is conceived and carried out in the analysed scene, requires a *rearrangement of the view*: a shift from calculating the numbers to seeing the interrelationships between them and recognising the pattern-forming rules. In this regard, the students in the analysed scene face a similar problem to the students in the studies done by Stevens and Hall (1998) and Lindwall and Lymer (2008). To properly see the linear functions, the graph, or, in this case, the number relations, the students must modify their visual and cognitive perception of the object of the discipline-specific knowledge they are dealing with. They must work it out: seeing how certain numbers are related to each other and recognising the pattern-forming rules "takes *work*" (Lindwall and Lymer 2008, 218; emphasis added), work consisting of the reflexive coordination of the material properties of the discipline-specific object of knowledge and the interactive practices of the students. This work is, of course, directed and controlled by the teacher, who sets the object and the goal of discovery and to some extent guides the students' attention. However, unlike the situation in Lindwall and Lymer's study and the tutoring episodes described by Stevens and Hall, here the instruction is largely "delegated" to the object of discovery itself. There are only few direct instructions from the teacher in the scene, and these come after the students have already moved from calculating the numbers to seeing the relationships between them and are mostly aimed at differentiating the students' ways of seeing and describing. The most instructive work is "done" by the arithmetic triangles, whose structure and visual properties create a "phenomenal field" (Lynch and Macbeth 1998, 277) of perception through which the students' discovery work is guided. They are instructive because, as Macbeth puts it, "they afford our interrogation to discover what they have to show us about 'nexts'" (2014, 307, n. 7).

Another constitutive feature of the students' discovery work is its embeddedness in the institutional and social setting of the classroom. As the students work in pairs and this "pair work" is situated in the social environment of the classroom—done with and among others in the class—it has a witnessable character and is produced as such. There are numerous moments in the investigated lesson when the students produce their claims of discovery, announcements of "getting it done," or comments on the "easiness" of the task, and these are recognisably *socially* oriented acts, addressed to other students or to the teacher. Since the organisation of students' work as "pair work" implies an expectation of "doing it together," it may also pose specific challenges for how this work is accomplished. For instance, while the two students in the episodes analysed displayed a clear orientation towards collaboration and were attempting to do their work as a team, there were also pairs of students in which the expectations of "working together" and "helping" were subject to negotiation or rejection. In this light, it may be worthwhile for further studies to analyse in detail how this social dimension of students' work in the classroom is intertwined with its subject-specific dimension, particularly students' engagement with discipline-specific knowledge as an object of learning and instruction.

As has been shown, the students' discovery is accomplished as an activity structured around a clear orientation towards the completion of a task. The drive towards *getting the task done* characterises the work of almost all students in the studied lesson (albeit with varying explicitness). I propose that, for students, this orientation is a reasonable way of carrying out their discovery work, as this work is set up as an educational school assignment. I will return to this idea when discussing how students' discovery work relates to learning.

Students' Discovery Work and Its Relation to Learning

What the students do in the lesson under consideration is explicitly framed as discovery work; to recall, the teacher formulated the tasks for the class using the words "maths research" and "to discover." As this discovery work is, however, situated in a school setting, it is closely tied to the institutional concerns of that setting—the educational goals of enabling the acquisition of (new) knowledge and facilitating students' learning. Where concerns about learning and knowledge acquisition are at stake, there is always an inherent normative component to what participants do. Whether in schools or other institutional settings, knowledge is always subject to evaluation and control measures that raise questions about appropriate and inappropriate ways of doing. In this respect, one could also ask how what the students do in the analysed scene can be considered both discovering and learning some mathematical phenomena. Or, to put it another way, what does the present analysis reveal about the students' discovery work and learning and their relation to each other?

As the analysed episodes only show a small fragment of a single lesson, namely the students' work on the first task of a mathematical assignment, it remains an open question what exactly the students have learned here. It is difficult to say with certainty whether the students have understood the underlying pattern that describes the relationship between the numbers in the arithmetic triangles they have discovered. In particular, it is not clear whether the two students in the analysed scene also realise the relationship between the inner and outer numbers when they formulate their description of the rule referring to the outer numbers through analogy with the rule referring to the inner numbers. If we were to look at the further course of the lesson, we could observe that at least one of the two students, Sm02, during the whole class discussion, can formulate an adequate description of the relationship between the inner numbers and the outer numbers. Specifically, he can explain that the outer numbers always double when the inner numbers increase by a certain numerical value. By the end of the lesson, he can also offer a reasonable explanation as to why this is so. In relation to the episodes analysed above, however, our analytical inferences about what the students have learned are limited. Yet, what we can observe here is a recognisable discovery effort on the part of the students. We can thus say with certainty that the students arrive at a certain insight by carrying out their exploration of the constitutive properties of the arithmetic triangles and by explicating and describing the relations between the numbers—that is, by accomplishing a set of activities that allow what was previously unknown to become known. In this respect, the students acquire new knowledge and thus *do* learn something new about number relations.

There are two clearly recognisable orientations in the way the students accomplish their discovery work, as discussed above: the orientation towards pursuing the discovery of the arithmetic triangles and figuring out the puzzle on the one hand and the orientation towards getting the task done on the other. The two orientations "interfere" (Breidenstein 2021) to some extent, as the students stop their exploration as soon as they feel they have found something out and signal to the teacher that they are done. One could thus likely say that the students are not truly interested in discovery but in fulfilling the instructional task. From this perspective, the students' orientation towards *getting a task done* may be critically treated as "doing the lesson" (Jimenez-Aleixandre, Bugallo Rodríguez, and Duschl 2000), focusing more on "following the instructions and satisfying the teacher than on the substance of the ideas" (Berland et al. 2012, 72).

However, when discussing how students do their discovery work, it is important to bear in mind that what students are doing is not only discovery, nor purely engaging with discipline-specific ideas. Rather, they are completing an educational assignment in school mathematics. For students, their discovery *is* a lesson task, and from this perspective it is reasonable behaviour to be oriented towards this task as a matter of instruction. As Macbeth (2002, 382) notes, the problems and activities of the participants in the lessons are primarily practical and only secondarily discipline-specific. Thus, when the students stop their exploration and signal to the teacher that they are done, they display their orientation to the practical relevancies of doing the work of accomplishing a mathematical assignment. In fact, the completion of a particular task is an assignment-relevant action for which the students can be held accountable, as this assignment consists of several pre-structured task steps clearly defined by the teacher. Thus, when the students terminate their discovery work, they do not do so at any arbitrary moment; they stop at the point when they feel they have fulfilled the given task steps—in this case, solving a puzzle and describing what they have noticed. The way in which the students do so is not merely a formal "going through" of the task steps but a recognisably meaningful engagement with the subject matter content. For example, when the students, committed by the teacher to further exploration of the arithmetic triangles, have difficulty in finding out what characterises the relationship between the outer numbers (excerpt 4), they pursue their discovery until they have figured out the rule (excerpt 5).

In this light, it is important to consider students' discovering and learning activities in relation to the specific concerns of the setting in which these activities take place and the interactive work in and through which they are constituted. Beyond the theoretical conceptualisations of what "learning" and "discovery" should be, they remain situated in the practical circumstances of the interactional practices of students and teachers, who mark and represent what they are doing as discovery-related and learning-relevant activities for themselves and others. From this perspective, "learning" must be understood as a matter of participants' practical concerns. That is, the issues of knowledge, competence, and understanding must be considered and analysed in relation to the practical concerns of those involved, teachers and students, as they carry out their everyday activities in the classroom according to the needs of the situation.

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APPENDIX: TRANSCRIPTION CONVENTIONS

?	slightly rising intonation
.	slightly falling intonation
,	continuation of tone
!	animated and emphatic tone
-	cut off of prior word or sound
=	no gap between two turns
Ja::	prolongation of sound
AUSSENzahlen	emphasis, either through increased volume or higher pitch
(.)	a pause of one second
(2.0)	a pause of two seconds
[...]	simultaneous talk
(und)	unclear or probable talk
(*inaudible*)	a stretch of talk that is unintelligible to the researcher

Multimodal details have been transcribed according to conventions developed by Mondada (2018):

*	gestures and descriptions of embodied actions are marked by identical symbols (one symbol per participant) and synchronised with talk
*	for gestures done by the teacher Tf
+	for gestures done by the student Sm02
•	for gestures done by the student Sm18

Other symbols:

↑	student raises his/her hand
↓	student lowers his/her hand
Δ	arithmetic triangle(s)
ws	worksheet
bb	blackboard

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